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An Analytic Model for Runoff and Runoff Temperature from a Paved Surface

by

William R. Herb, Ben Janke, Omid Mohseni, and Heinz G. Stefan

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Abstract

Existing simplified runoff models such as SCS synthetic hydrographs give some ability to predict surface runoff, but are generally developed for larger watersheds, and do not necessarily represent the actual variation in flow rate with varying precipitation rate. For the purposes of simulating runoff rate and runoff temperature from small parcels of land, a new runoff model was developed based on Manning's equation. The runoff model is analytical and spatially integrated (zero-dimensional), in that flow depth, flow rate and runoff temperature are computed at one point, the outlet. By taking into account expected variations in the upstream flow depth, the model closely matches the simulations results of a 1D kinematic wave model. The analytic runoff model was coupled to a 1D soil temperature and moisture model, to enable simulation of infiltration, runoff rate and runoff temperature.

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NOTATIONS AND UNITS

i = rainfall intensity [cm/h]
L = length of parking lot, or section of lot being modeled [m]
L_e = length of fully-developed flow on lot surface [m]
n = surface roughness, used in Manning's Equation []
q = runoff flow rate [m^2/s or m/s], also specific humidity (Eq. B.12) [kg_w/kg_a]
q_{ro} = per-width runoff flow rate at the lot outlet [m^2/s]
S_0 = slope of the bottom of the channel / parking lot [dimensionless]
t = time [h]
t_e = time to equilibrium [h]
T_{ro} = temperature of the runoff on the pavement surface [°C]
v = runoff volume per unit width [m^2]
x = horizontal spatial coordinate [m]
y = depth of runoff or standing water on pavement surface [cm]

1. INTRODUCTION

Runoff models such as the kinematic wave model are commonly used to simulate the temporal and spatial distribution of precipitation runoff over land surfaces. A one-dimensional (1-D) kinematic wave model was developed at SAFL to simulate runoff depth and flow rate as a function of distance (Janke et al. 2006). That kinematic wave runoff model was coupled to a soil heat transfer model to predict runoff temperature. The resulting coupled hydro-thermal model is computationally intensive, and simulation results indicate that runoff temperature varies much more strongly with time than distance. To analyze runoff from multiple sub-watersheds with a variety of land surfaces, the 1-D hydro-thermal model becomes too computationally intensive. Existing runoff models such as synthetic hydrographs give some ability to predict surface runoff, but are generally developed for larger watersheds, and do not necessarily represent the actual variation in flow rate with varying precipitation rate. For the purposes of this study, a new runoff model was developed, based on Manning's equation. The runoff model is analytic and zero-dimensional, in that flow depth and rate are computed at one point, the outlet of the sub-watershed. By taking into account expected variations in the upstream flow depth, the analytic model can be closely matched with simulations results of the 1-D kinematic wave model.

2. ANALYTIC MODEL FORMULATION FOR CONSTANT RAINFALL INTENSITY

The analytic runoff model is based on Manning's equation, which gives a relationship between runoff depth (y) and flow rate per unit width (q) for a land surface with a slope (S) and a Manning's roughness (n):

(1) $$q = \frac{S_0^{1/2} y^{5/3}}{n}$$

Based on results from the kinematic wave model (Janke et al. 2006), the runoff surface is divided into a length with fully developed flow (L_e) and a length with constant runoff depth, y. For a surface of total length L (Figure 1), the time for the flow to reach equilibrium can be estimated from Manning's equation as (Mays 2001)

(2) $$t_e = \frac{n L}{S_0^{1/2} y^{2/3}}$$

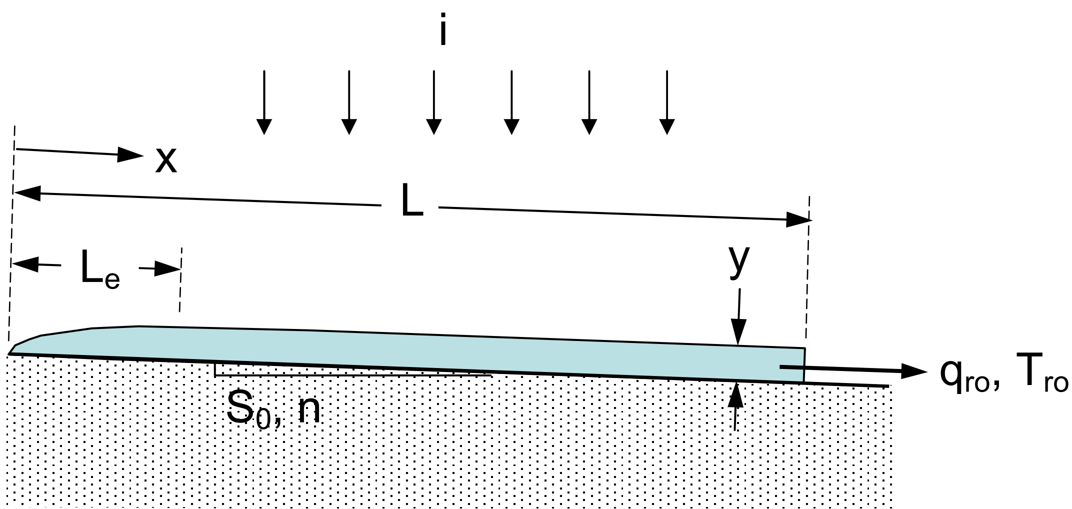

Figure 1. Schematic diagram of the parameters used in the analytic runoff model.

For rainfall intensity i, the equilibrium flow rate, q, at any point, x, can be determined from a mass balance as:

(3) $q = i \cdot x$

Equations 1 and 3 may be combined to yield an expression for y(x) at steady state flow conditions:

(4) $$y(x) = \left(\frac{i\,x\,n}{S_0^{\;1/2}} \right)^{3/5}$$

Combining Equations 2 and 4 yields to the following equation for the time to equilibrium, also given in Wilson (2004):

(5) $$t_e = \left(\frac{nL}{i^{2/3} S^{1/2}} \right)^{3/5}$$

The time to equilibrium is estimated based on velocities at the equilibrium flow rate, so that some error in this estimate can be expected. For $t<t_e$, an expression for the equilibrated runoff length L_c for some time t may be found by rearranging Equation 5:

(6) $$L_e = \frac{S^{1/2}\, i^{2/3}\, t^{5/3}}{n}$$

The runoff volume per unit width (v) may be expressed as:

(7) $$v = \frac{5}{8} h\, L_c + y\left(L - L_e\right) = y\left(L - \frac{3}{8} L_e \right)$$

The factor of 5/8 is found by integrating Equation 4 over the length L_e to find an average depth. The rate of change of the runoff volume can be expressed in difference form as:

(8) $$v(t + \Delta t) = v(t) + i\,L\,\Delta t - \frac{\Delta t}{2}\left(q(t) + q(t + \Delta t)\right)$$

Using Manning's equation and Equation 7, the runoff volume can be expressed in terms of the flow rate as:

(9) $$v(t + \Delta t) = q(t + \Delta t)^{3/5} \left(\frac{n}{\sqrt{S_0}} \right)^{3/5} \left(L - \frac{3}{8} L_e (t + \Delta t) \right)$$

Substituting Equation 9 into Equation 8:

(10) $$q(t + \Delta t)^{3/5} \left(\frac{n}{\sqrt{S_0}} \right)^{3/5} \left(L - \frac{3}{8} L_e (t + \Delta t) \right) + q(t + \Delta t)\frac{\Delta t}{2} = v(t) + i\,L\,\Delta t - q(t)\frac{\Delta t}{2}$$

Equation 10 is non-linear in the unknown flow rate, $q(t+\Delta t)$. To linearize the equation, the following Taylor series expansion is used:

(11) $q(t+\Delta t)^{3/5} \approx q(t)^{3/5} + \frac{3}{5} q(t)^{-2/5} \left(q(t+\Delta t) - q(t)\right)$

Substituting Equation 11 into 10 yields a linear difference equation for the new flow rate, q(t+Δt):

(12) $$q(t+\Delta t) = \frac{v(t) + i\,L\,\Delta t - q(t)\frac{\Delta t}{2} - \frac{2}{5}\left(\frac{n}{\sqrt{S_0}}\right)^{3/5}\left(L - \frac{3}{8}L_e(t+\Delta t)\right) q(t)^{3/5}}{\frac{\Delta t}{2} + \frac{3}{5}\left(\frac{n}{\sqrt{S_0}}\right)^{3/5}\left(L - \frac{3}{8}L_e(t+\Delta t)\right) q(t)^{-2/5}}$$

Analysis of a rainfall event with constant intensity may then proceed as follows:

1) For the start of the rainfall event, the runoff volume is estimated as v(0) = i·L·Δt, the runoff depth as i·Δt, and the flow rate at the end of the first time step is calculated using Manning's equation (Equation 1).
2) For each subsequent time step during the rainfall event, Equation 6 is used to calculate L_e(t+Δt), Equation 12 is used to calculate q(t+Δt), and Equation 8 is used to calculate v(t+Δt). L_e is not allowed to exceed L
3) At the end of the rainfall event, the calculation continues with i=0 in Equation 12. If L_e is less than L, i.e. the rainfall event is shorter than the time to equilibrium, L_e is assumed to follow Equation 6 as if the rainfall continues at the same intensity.

3. ANALYTIC RUNOFF MODEL FORMULATION FOR UNSTEADY RAINFALL EVENTS

For rainfall events with time varying intensity, the algorithm is modified as follows. As the rainfall intensity changes, the runoff thickness, volume and equilibrated runoff length (L_e) should vary continuously, i.e. with no discontinuity. However, if Equation 6 is used to calculate L_e, a discontinuous rainfall intensity will result in a discontinuous value of L_e and of the runoff volume. To correct this, a time displacement is introduced, such that L_e is continuous. For the time t_1, where the rainfall intensity changes from i_1 to i_2 (Figure 2), a new time t'_1 is calculated based on Equation 6:

$$(13) \quad L_e = \frac{S^{1/2}\, i_1^{\;2/3}\, t_1^{\;5/3}}{n} = \frac{S^{1/2}\, i_2^{\;2/3}\, t'^{\;5/3}_1}{n}$$

The time t'_1 represents a new time base for the new rainfall intensity, such that the equilibrium length L_e is continuous. Solving for t'_1:

$$(14) \quad t'_1 = t_1 \left(\frac{i_1}{i_2} \right)^{2/5}$$

For the new rainfall intensity i_2, the equilibrated runoff length is calculated using Equation 15, and the flow calculation proceeds as before.

$$(15) \quad L_e = \frac{S^{1/2}\, i_2^{\;2/3}}{n} \left(t'_1 + (t - t_1) \right)$$

where t is the actual time from the start of the event.

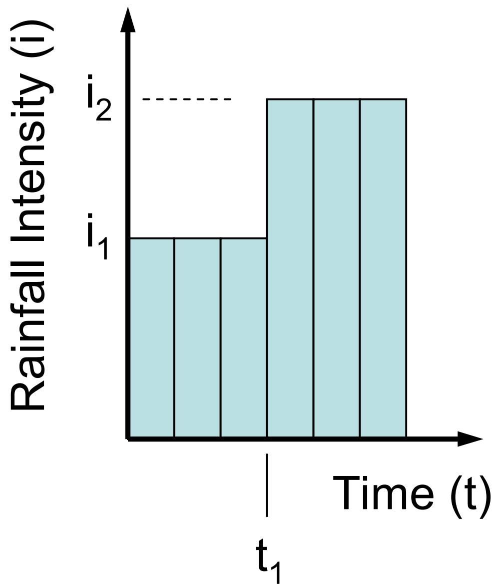

Figure 2. Time and rainfall parameters for unsteady precipitation event

4. RUNOFF MODEL EVALUATION

Presently, suitably detailed runoff data are not available for model verification and calibration. To evaluate the analytic runoff model, simulated runoff time series from the 1-D model are compared to results from the 1-D kinematic wave model. Runoff was also computed using the SCS synthetic hydrograph method (Mays 2001), using Equation 5 to calculate the time of concentration, except where noted.

The runoff response of three cases with constant rainfall intensity is shown in Figures 3, 4 and 5. Figure 3 presents a case where the rainfall event is much longer in duration (2 hours) than the equilibrium time for the surface (200 seconds), e.g. a long duration rainfall event running off a road or small parking lot. For this case, most of the runoff takes place in equilibrium, and all three models (analytic, 1-D, SCS) give reasonable results.

Figure 4 gives the computed runoff response to a steady rainfall event with very low intensity (i=0.5 cm/hour) over a larger paved area (L=400 m). Compared to the kinematic wave results, the analytic model has somewhat different behavior later in the event. This is due to the way in which L_e is estimated after the rainfall stops in the analytic model, i.e. L_e is calculated as if the rainfall continues. The SCS method using Equation 5 to calculate t_e is substantially off compared to the 1-D and kinematic wave results. Also shown in the Figure 4 are results for the SCS method using the lag time method to estimate t_e (Mays 2001):

$$(16)\quad t_e = L^{0.8}(R+1)^{0.7} / \left(1140\ S_0^{\ 0.5}\right)$$

where R is the potential maximum abstraction, and the other terms are as defined previously.

The lag time method does not take into account the rainfall rate, so that for the low rainfall rate used in this example, the lag time method does not give a good estimate of the time of concentration, leading to additional error in the SCS runoff computation.

Figure 5 gives a case where the rainfall event shorter in duration (15 minutes) than the equilibrium time for the surface (20 minutes) with a relatively high intensity (i=5 cm/hour), e.g. a short duration rainfall event running off a large parking lot. For this case, the analytic model matches the kinematic wave results quite, including the peak flow and the time of peak. The SCS method hydrograph is moderately off in both the peak flow and the time of the peak. Figure 6 gives results of analytic runoff computations for varying time step, from 1 minute to 15 minute. The 1 and 3 minute simulations give very similar results. The 5 minute time step gives some change in the peak response, but still very similar rising and falling legs. The 15 minute time step gives rather crude results, but the numbers are still reasonable.

Similar results for the three methods were obtained for an unsteady rainfall event (Figure 6).

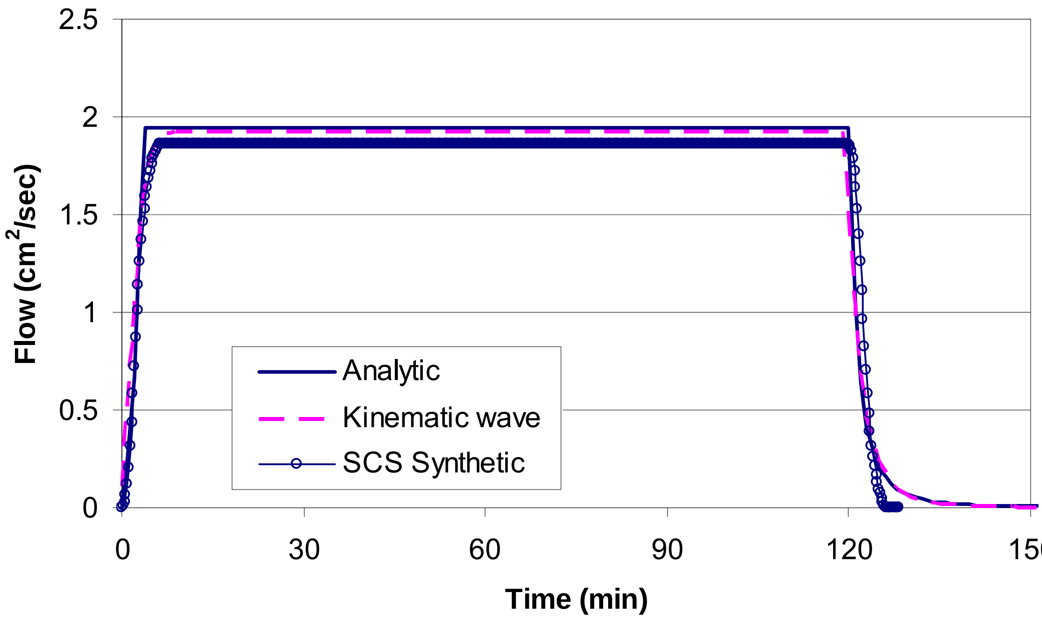

Figure 3. Runoff flow rate versus time for a steady rainfall event using the analytic model, the 1-D kinematic wave model, and the SCS synthetic hydrograph method. L=250 m, n=0.022, S=0.035, i=2.8 cm/hour, duration = 2 hours.

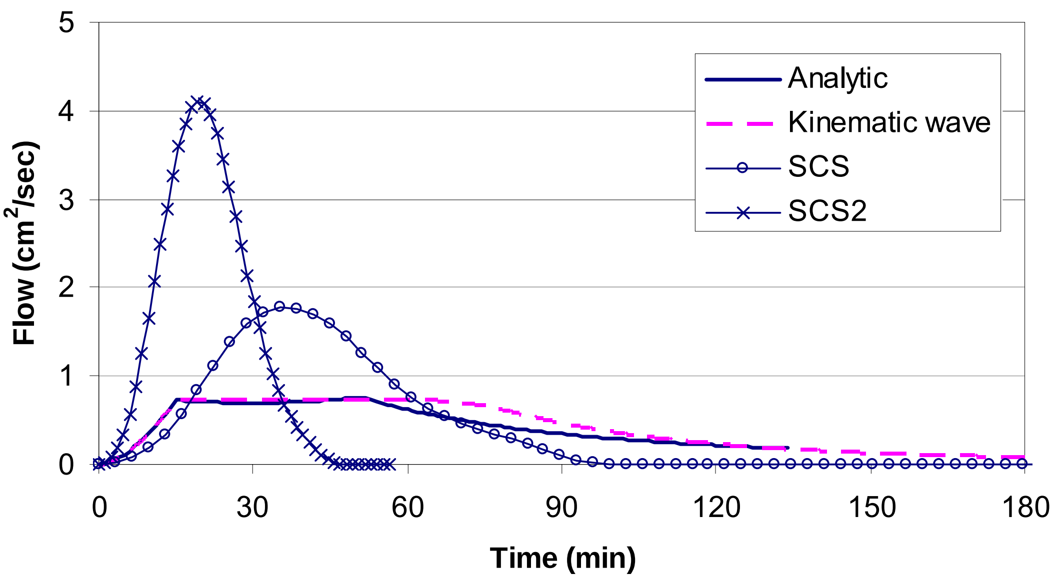

Figure 4. Runoff flow rate versus time for a steady rainfall event using the analytic model, the 1-D kinematic wave model, and the SCS synthetic hydrograph method. L=400 m, n=0.02, S=0.01, i=0.5 cm/hour, duration = 15 min. Results are given for the SCS method using Equation 5 to calculate t_c (SCS) and using the lag time method (Equation 16) to calculate t_c (SCS2).

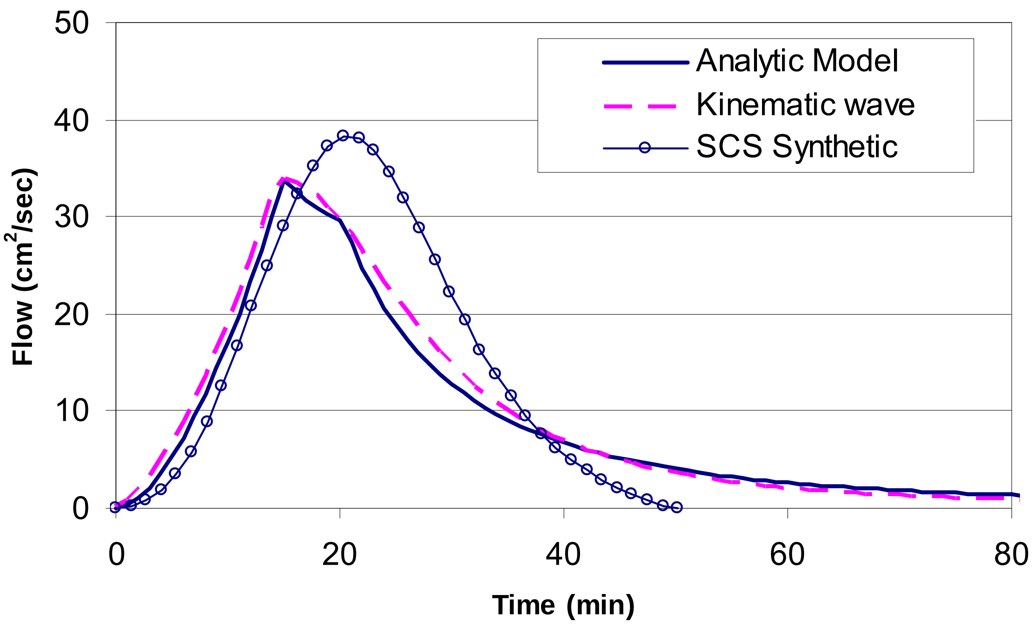

Figure 5. Runoff flow rate versus time for a steady rainfall event using the analytic model, the 1-D kinematic wave model, and the SCS synthetic hydrograph method. L=400 m, n=0.02, S=0.01, i=5 cm/hour, duration = 15 min.

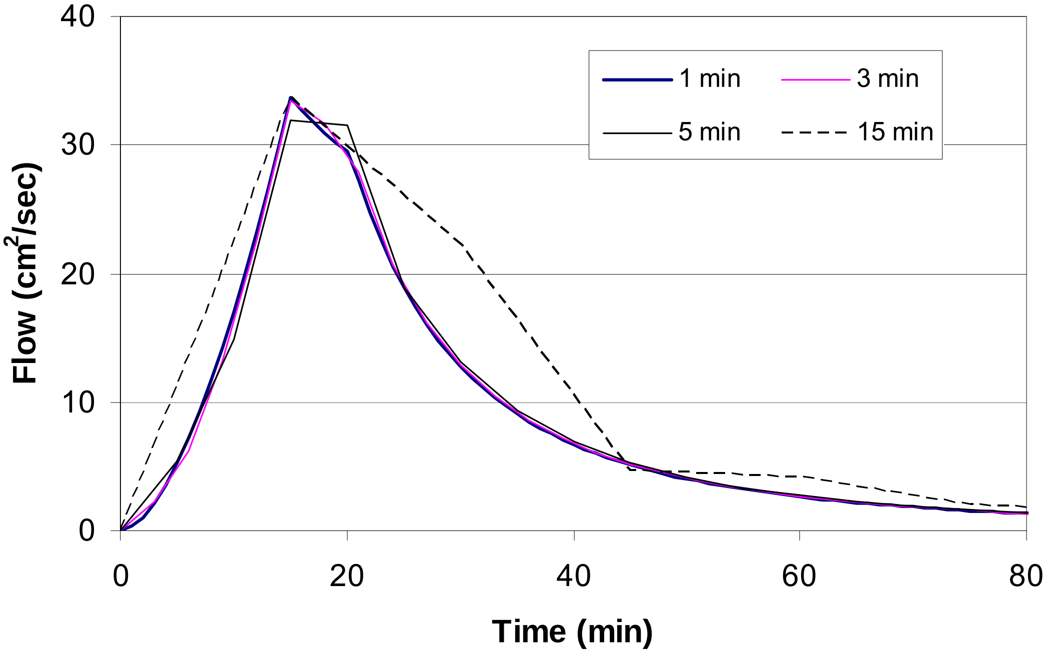

Figure 6. Runoff flow rate versus time for a steady rainfall event using the analytic model, the 1-D kinematic wave model, and the SCS synthetic hydrograph method. L=400 m, n=0.02, S=0.01, i=5 cm/hour, duration = 15 min.

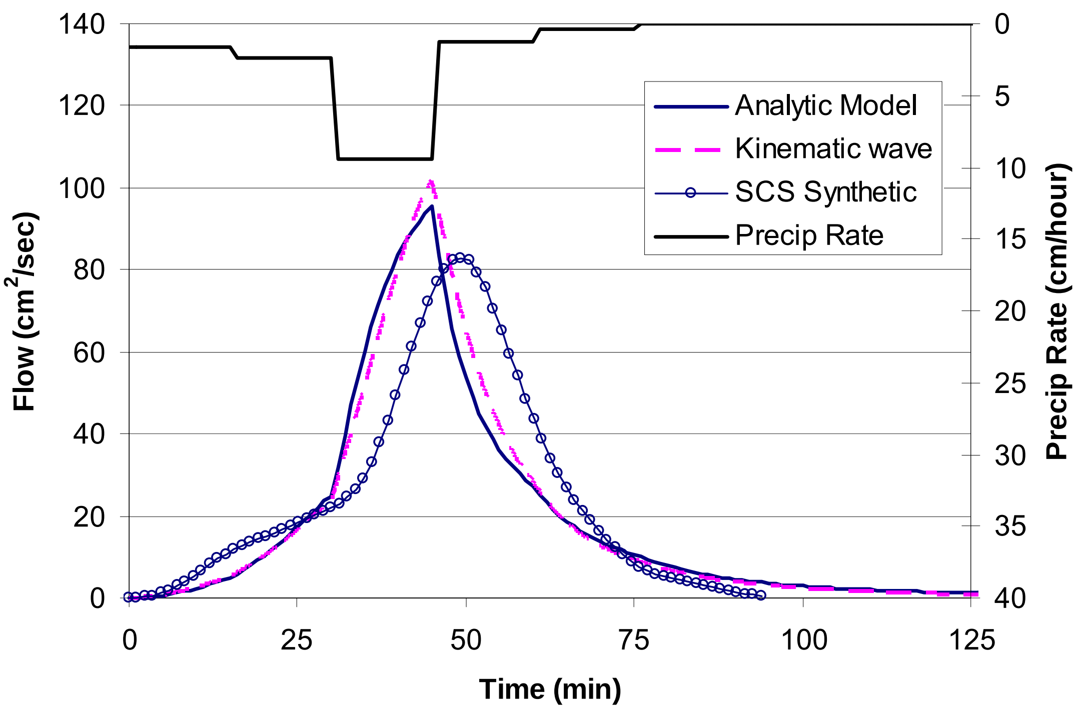

Figure 7. Runoff flow rate versus time for an unsteady rainfall event, using the analytic model, the 1-D kinematic wave model, and the SCS synthetic hydrograph method. L=400 m, n=0.02, S=0.01.

5. COMBINATION OF RUNOFF MODEL WITH A 1-D HEAT TRANSFER MODEL

To enable prediction of both runoff flow rate and runoff temperature, the analytic runoff model described here was added to the previously developed 1-D heat transfer model described in a separate document (Herb et al. 2006). The 1-D runoff/heat transfer model calculates an estimated runoff temperature based on the temperature of the underlying land surface, as described in the Appendix. Heat transfer between the runoff and the land surface is calculated based on the rainfall depth during each time step, assuming the all of the rainfall runoff reaches thermal equilibrium with the land surface as it runs off. Stream-wise variation in the runoff temperature and the surface temperature is not taken into account in the 1-D model.

The computational sequence for wet weather proceeds as follows:

(1) The atmospheric fluxes (net radiation, convection, and evaporation) into the pavement are determined based on the current values of the air temp, solar, etc. and the old value of the pavement surface temp. Since the pavement is wet, the heat fluxes are calculated using surface properties of water, e.g. albedo and emissivity.

(2) The amount of heat flux from the pavement to the runoff is estimated, based on the depth of new precipitation, the depth of penetration into the pavement, the pavement temperature from the previous time step, and the current rainfall temperature.

(3) The runoff heat flux from (2) is added to the atmospheric heat flux from (1), and the total heat flux is applied as a boundary condition to the soil temperature model to calculate the new pavement/soil temperature profile. The runoff temperature is assumed equal to the pavement surface temperature.

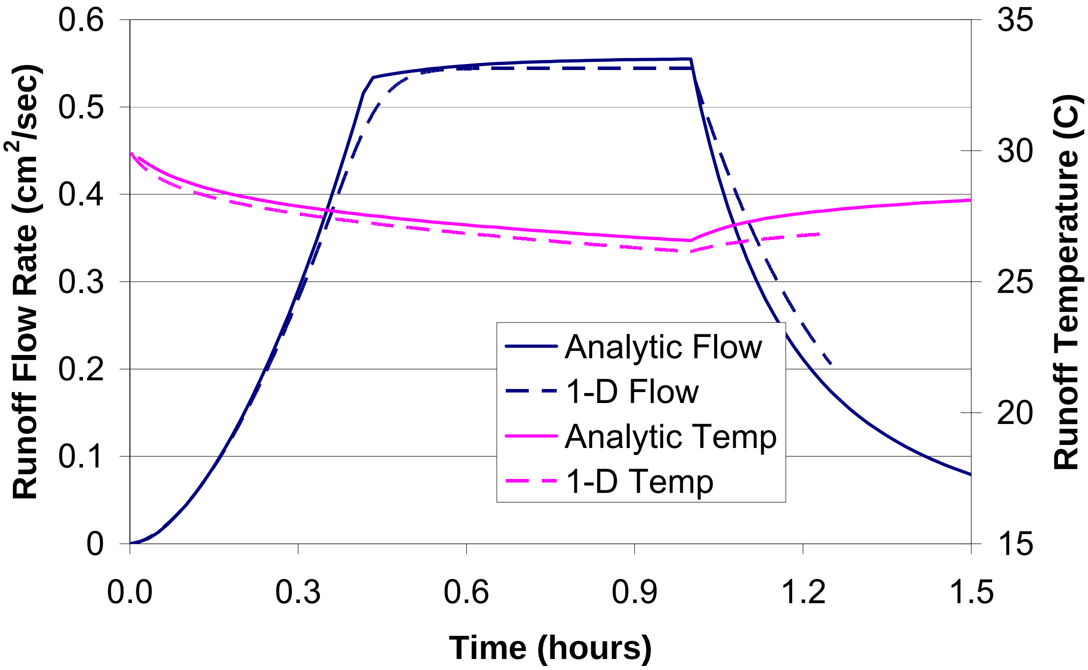

Figure 8. Analytic and 1-D simulated runoff flow rate and temperature for runoff from a low roughness or high slope surface. L=100 m, $\sqrt{S_0}\,/\,n = 8.5$, i=0.8 cm/hour, duration = 1 hour, initial surface temperature = 30 ºC, rainfall temperature = 20 ºC.

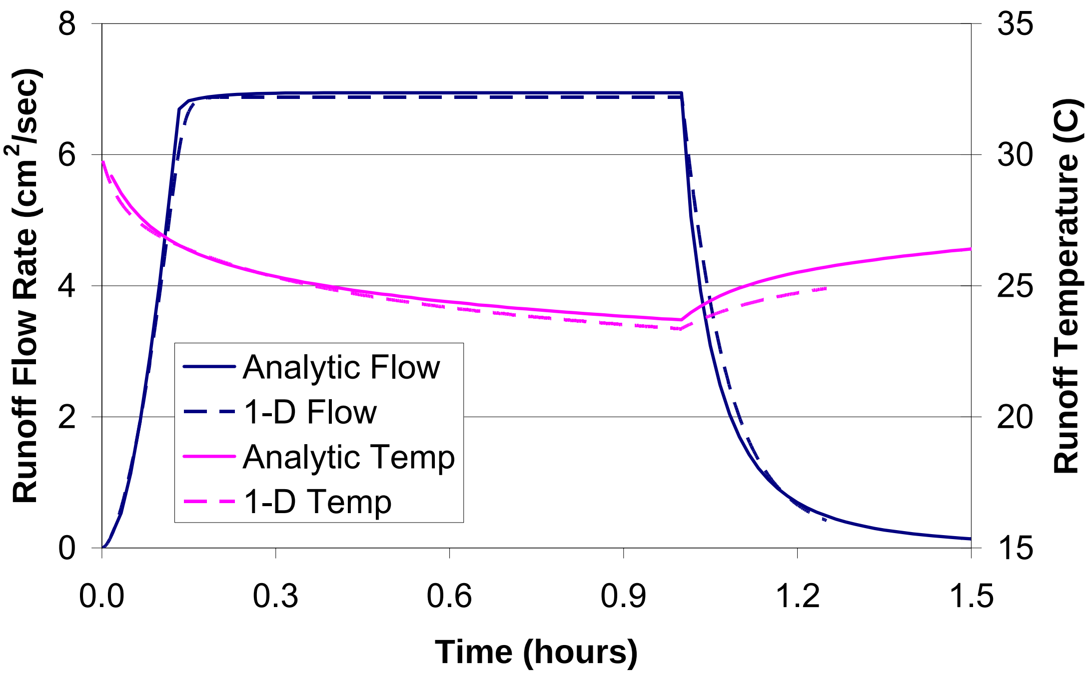

Figure 9. Analytic and 1-D simulated runoff flow rate and temperature for runoff from a high roughness or low slope surface. L=25 m, $\sqrt{S_0}\,/\,n = 0.65$, i=2.5 cm/hour, duration = 1 hour, initial surface temperature = 30 ºC, rainfall temperature = 20 ºC.

6. CONCLUSIONS

The analytic runoff model described here compares favorably to the 1-D kinematic wave model for predicting the time variation of runoff rate and temperature from impervious surfaces such as parking lots. The analytic model appears to provide a numerically efficient method for simulating runoff from multiple, small sub-watersheds contained in a commercial or residential development. Further work is required to evaluate the model for other land uses such as agricultural.

ACKNOWLEDGEMENTS

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APPENDIX. FORMULATION FOR MODELING THE HEAT FLUX BETWEEN RUNOFF AND GROUND

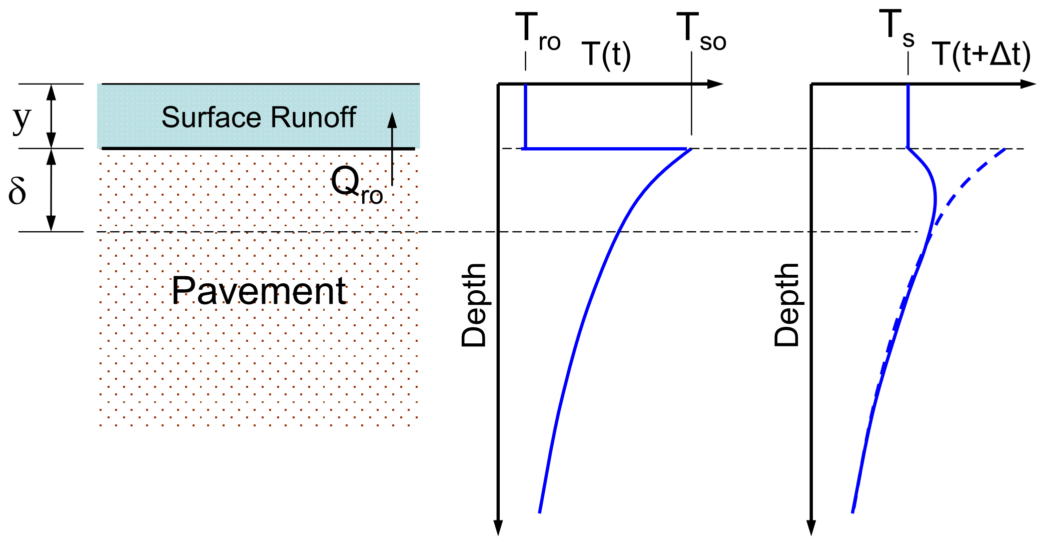

Figure A-1. Schematic of the formulation for heat transfer between surface runoff and the underlying pavement. Example temperature profiles show the temperatures in the pavement prior to a rainfall event (T(t)) and after a rainfall event (T(t+Δt)), where the initial surface temperature (T_{so}) has equilibrated with the initial runoff temperature (T_{ro}) to yield the new surface and runoff temperature (T_s).

The heat transfer model assumes that the surface runoff, assumed to be initially at dew point temperature, and pavement surface must equilibrate to the same temperature over the time step (Δt), as shown in Figure A-1. To achieve this equilibrium, a convective heat flux Q_{ro} can be calculated, which draws heat out of a thin layer of pavement at the surface with thickness δ. The size of δ can be estimated from the thermal properties of the pavement and the time step, based on analytic solutions for heat conduction into an infinite slab subject to a change in surface temperature (Eckert and Drake, 1972).

(A-1) $\delta = \sqrt{4\alpha\Delta t}$

where α is the thermal diffusivity and Δt is the time step. For a time step of 15 minutes and a thermal diffusivity of 4 x 10^{-6} m^2/s, δ = 12 cm. An equation for the heat balance for the water film layer and the pavement may then be written as:

(A-2) $Q_{ro} = y(\rho c_p)_w (T_s - T_{ro}) = -\frac{\delta}{2}(\rho c_p)_p (T_s - T_{so})$ (J/m^2)

where $(\rho c_p)_p$ and $(\rho c_p)_w$ are (density · specific heat) for the pavement and water, and the factor ($\delta/2$) takes into account that the temperature change in the pavement decreases with depth, whereas the temperature change in the water film as assumed to be uniform across the thickness y. If the dew point temperature and the initial surface temperature are known, Equation A-2 can be solved for T_s and Q_{ro}.

(A-3) $$T_s = \frac{T_{ro} + \beta T_{so}}{1+\beta}, \text{ where } \beta = \frac{\delta(\rho c_p)_p}{2\,y(\rho c_p)_w}$$

(A-4) $$Q_{ro} = h(\rho cp)_w (T_{so} - T_{dp})\left(\frac{\beta}{1+\beta}\right) \quad (\text{J/m}^2)$$

Using Equation A-4, the heat flux from the pavement to the runoff may be estimated for each time step based on the total precipitation in each time step, and is simply added to the atmospheric heat flux components. At any given time, the actual runoff depth is typically much less than the total precipitation depth, but the entire precipitation depth can still be expected to equilibrate with the pavement surface, so that applying the entire precipitation depth over the time step is equivalent to applying the same amount as a series of thin layers over shorter time steps. For the one-dimensional model, there is no consideration of lateral variations in runoff temperature from upstream to downstream points.